

Operations Research
Course Number: 26:711:651
Course Title: Linear Programming

COURSE DESCRIPTION

The aim of this course is to introduce graduate students to linear programming and its extensions with an emphasis on the mathematical formulations, algorithms and solutions for practical problems arising in business research and operations research including supply chains, network science, marketing and finance. The class will also include programming exercises with AMPL and Excel software for formulating and solving real world problems.

COURSE MATERIALS

Required texts:

- M.S. Bazaraa, J.J. Jarvis, H.D. Sherali, *Linear programming and network flows*, 4th Edition, Wiley, 2010.

(This is our main text).

- R. Fourer, D. Gay, B. Kernighan, *AMPL: A Modeling Language for Mathematical Programming*, 2nd Edition, Boyd & Fraser Publishing Company, 2002.

(This is important for learning AMPL, a modeling language for linear and non-linear programming. It is available in pdf format from the site <https://ampl.com/resources/the-ampl-book/>).

- Check Canvas and your official Rutgers email account regularly.

Learning Management System: Canvas

LEARNING GOALS AND OBJECTIVES

This course is designed to help students develop skills and knowledge in the following areas:

- *Developing skills: Mathematical background for linear programming, ability to interpret, formulate, solve and analyze linear programs arising in practical applications.*

- *Developing knowledge base: Students will get more familiar with linear programming tools so that they can use it in their own research both in social sciences and applied sciences. The students will have a good understanding of both the theory and practice for solving linear programming problems arising in the business research.*

- Students who complete this course will demonstrate the following:

- *Ability to apply linear programming techniques for solving and modeling some fundamental decision making problems arising in the daily business life.*

- Students develop these skills and knowledge through the following course activities and assignments:

- Students will complete homework assignments, a midterm and a final exam. Lectures will be interactive with students, allowing them to interact with their peers in collaborative class discussions.

PREREQUISITES

A good knowledge of undergraduate level linear algebra and calculus techniques for optimization such as matrix computations, optimality conditions and Lagrange multipliers.

- Familiarity with at least one software tool that can be used for numerical computations (such as Excel, R, MATLAB, Python or equivalent).

ACADEMIC INTEGRITY

I do NOT tolerate cheating. Students are responsible for understanding the RU Academic Integrity Policy <http://academicintegrity.rutgers.edu/>

I will strongly enforce this Policy and pursue *all* violations. On all examinations and assignments, students must sign the RU Honor Pledge, which states, “On my honor, I have neither received nor given any unauthorized assistance on this examination or assignment.” I will screen all written assignments through *SafeAssign* or *Turnitin*, plagiarism detection services that compare the work against a large database of past work. Don’t let cheating destroy your hard-earned opportunity to learn. See business.rutgers.edu/ai for more details.

ATTENDANCE AND PREPARATION POLICY

- Expect me to attend all class sessions. I expect the same of you. If I am to be absent, my department chair or I will send you notice via email and Canvas as far in advance as possible. If you are to be absent, report your absence in advance at <https://sims.rutgers.edu/ssra/>. If your absence is due to religious observance, a Rutgers-approved activity, illness, or family emergency/death and you seek makeup work, also send me an email with full details and supporting documentation within 3 days of your first absence.

- For weather emergencies, consult the campus home page. If the campus is open, class will be held.

- Expect me to arrive on time for each class session. I expect the same of you.

- Expect me to remain for the entirety of each class session. I expect the same of you.

- Expect me to prepare properly for each class session. I expect the same of you. Complete all background reading and assignments. You cannot learn if you are not prepared. The minimum expectation is that for each class, you have prepared by studying for at least twice as many hours.

- Expect me to participate fully in each class session. I expect the same of you. Stay focused and involved. You cannot learn if you are not paying attention.

You are expected to attend all classes. Participation will be required.

CLASSROOM CONDUCT

Do not call out in class. If you have a question, raise your hand.

- No food or drink in class.
- No side conversations or use of cell phones in class.
- Use the bathroom before class to avoid the necessity of bathroom breaks.

EXAM DATES AND POLICIES

There are **two** exams in this course. The exam dates are:

- Midterm exam:
- Final exam:

During the exam, the following rules apply:

- If you have a disability that influences testing procedures, provide me an official letter from the Office of Disability Services **at the start of the semester**.
- No cell phones or other electronics are allowed in the testing room.
- You must show a valid Rutgers photo ID to enter the room and to turn in the exam.
- Alternate seating; do not sit next to another student or in your usual seat. A new seat will be assigned on a random basis.
- Use the bathroom prior to the exam start; bathroom breaks, if essential, will be escorted.
- Your exam will not be accepted unless you sign the Honor Pledge

Make-up exam policy:

- **Allowances for make-up exams** “Make-up” exams are allowed only for those students whose absence on a class exam date was due to a legitimate illness or emergency (i.e., circumstances beyond their control). MSIS Department Coordinator Office ultimately determines what does or does not constitute a “legitimate” illness/emergency.

- **Procedures for obtaining authorization to take a make-up exam:** If your absence is due to illness, the MSIS Department Coordinator Office will require you to provide them with a document from your doctor indicating that you were indeed sick that day. If you do not provide the MSIS Department Coordinator Office with a doctor's note, they cannot in turn provide me with the proper authorization to allow a make-up exam.

GRADING POLICY

There will be 8-10 homework assignments, issued on after class. Homework assignments will include programming exercises as well as conceptual questions. Each assignment should be submitted online on Canvas, in any legible format (scans of handwritten solutions are fine). All homework assignments are to be completed on your own. You are welcome to discuss homework problems with me during office hours. Should a student copy the homework of another student, both of them will get only half points.

All homework assignments and exams will be graded on a scale from A to F. Your final grade will be a weighted average taken according to the following weights.

30% Homework Assignments

30% Midterm Exam

40% Final Exam

Your final grade is not subject to negotiation. If you feel I have made an error, submit your written argument to me within one week of receiving your final grade. Clarify the precise error I made and provide all due supporting documentation. If I have made an error, I will gladly correct it. But I will adjust grades only if I have made an error. I cannot and will not adjust grades based on consequences, such as hurt pride, lost scholarships, lost tuition reimbursement, lost job opportunities, or dismissals. Do not ask me to do so. It is dishonest to attempt to influence faculty in an effort to obtain a grade that you did not earn, and it will not work.

COURSE SCHEDULE

Topic

Formulations and definitions.

Topics: Introduction to linear programming (LP), history of LP, some real world problems including investment under taxation, diet problem, applications to manufacturing and scheduling, introduction to software (AMPL and Excel)

Reading: Bazaraa et al (Chapter 1), AMPL reference book (Chapter 1)

Review of linear algebra

Topics: Quick review of basic facts from matrices: block matrices, transpose, inverse, determinant, Gaussian elimination and LUP factorization, Cholesky factorization, matrix notation for linear programs.

Reading: AMPL book (Chapter 2), Bazaraa et al (Chapter 2)

Review of convex sets and functions, polyhedra

Topics: Convex sets and function, separation theorem, convex cones, hyperplanes, polyhedra, faces, facets, edges, and extreme points and rays, Caratheodory's theorem. Convex polytopes, convex hulls.

Reading: AMPL book (Chapter 2), Bazaraa et al (Chapter 2)

The simplex method (part 1)

Topics: The basic feasible solutions and connection to extreme points, basic and non-basic variables, the simplex iteration, reduced costs and exchange of basic and nonbasic variables, optimality criterion. The tableau notation. Finding an initial basic solution, the Big M method.

Reading: Bazaraa et al (Chapters 3-4)

The simplex method (part 2)

Unbounded, and infeasible problems, and their detection degeneracy and non-unique solutions, possibility of cycling and methods to avoid it. The so-called revised simplex method (matrix formulation of the simplex method). The Simplex method for bounded variables. Farkas' Lemma via the simplex method.*

Reading: Bazaraa et al (Chapter 4-5)

Duality and complementary slackness

Topics: The dual problem, weak duality theorem, strong duality using the tableau. Dual of the diet and the transportation problems, economic interpretation of duality, the arbitrage theorem in finance and duality. Convex sets and functions, separation theorem, Farkas lemma, proof of duality theorem based on Farkas lemma.*

Reading: lecture notes, Bazaraa et al. (Chapter 6)

The dual simplex method

Topics: The dual simplex method. Complementary slackness theorem and its consequences, economic interpretation primal-dual simplex methods dual of general linear programs.

Reading: lecture notes, Bazaraa et al. (Chapter 6)

Computational complexity and Interior point algorithms

Topics: Polynomial complexity issues. Computational complexity of the simplex algorithm. Kachian's Ellipsoid algorithm.

Reading: Bazaraa et al. (Chapter 8)

Mid-term exam

Introduction to game theory

Topics: concept of matrix games, optimal strategies and the min-max theorem, connection to duality in linear programming Nash equilibrium, some economic applications.

Network flows

Topics: The general minimum cost network flow problem. Special cases such as the transshipment problem, transportation problem, maximum flow problem, assignment problem and shortest path problem. Interpretation of bases and pivots in network problems: spanning trees, and bases.

Reading: Bazaraa et al. (Chapters 9, 10 and 12), chapter 15 of AMPL book

The network simplex method

Topics: The network simplex method. Primal-dual simplex method and its application to network flow problems. The transportation problem, simplex and primal-dual simplex methods. Assignment problem and the Hungarian method maximum flow problem and Ford-Fulkerson augmenting path algorithm and Goldberg's push-relabel algorithm. Network flow problems with loss or gain on arcs or on nodes, applications to currency arbitrage. Dijkstra's shortest algorithm and dynamic programming and connection primal-dual method.

Reading: Bazaraa et al. (Chapters 9, 10 and 12), chapter 15 of AMPL book

Introduction to Integer programming

Topics: Integer Programming and mixed integer programming and contrast to linear programming. Fundamental difficulty of solving integer programs. The branch and bound method: A simple case: The knapsack problem. The branch and bound method. Applications.

Reading: Chapter 20 of AMPL book

Final exam

SUPPORT SERVICES

If you need accommodation for a *disability*, obtain a Letter of Accommodation from the Office of Disability Services. The Office of Disability Services at Rutgers, The State University of New Jersey, provides student-centered and student-inclusive programming in compliance with the Americans with Disabilities Act of 1990, the Americans with Disabilities Act Amendments of 2008, Section 504 of the Rehabilitation Act of 1973, Section 508 of the Rehabilitation Act of 1998, and the New Jersey Law Against Discrimination. More information can be found at ods.rutgers.edu.

[Rutgers University-New Brunswick ODS phone (848)445-6800 or email dsoffice@echo.rutgers.edu]

[Rutgers University-Newark ODS phone (973)353-5375 or email ods@newark.rutgers.edu]

If you are *pregnant*, the Office of Title IX and ADA Compliance is available to assist with any concerns or potential accommodations related to pregnancy.

[Rutgers University-New Brunswick Title IX Coordinator phone (848)932-8200 or email jackie.moran@rutgers.edu]

[Rutgers University-Newark Office of Title IX and ADA Compliance phone (973)353-1906 or email TitleIX@newark.rutgers.edu]

If you seek *religious accommodations*, the Office of the Dean of Students is available to verify absences for religious observance, as needed.

[Rutgers University-New Brunswick Dean of Students phone (848)932-2300 or email deanofstudents@echo.rutgers.edu]

[Rutgers University-Newark Dean of Students phone (973)353-5063 or email DeanofStudents@newark.rutgers.edu]

If you have experienced any form of *gender or sex-based discrimination or harassment*, including sexual assault, sexual harassment, relationship violence, or stalking, the Office for Violence Prevention and Victim Assistance provides help and support. More information can be found at <http://vpva.rutgers.edu>.

[Rutgers University-New Brunswick incident report link: <http://studentconduct.rutgers.edu/concern/>. You may contact the Office for Violence Prevention and Victim Assistance at (848)932-1181]

[Rutgers University-Newark incident report link: https://cm.maxient.com/reportingform.php?RutgersUniv&layout_id=7. You may also contact the Office of Title IX and ADA Compliance at (973)353-1906 or email at TitleIX@newark.rutgers.edu. If you wish

to speak with a staff member who is confidential and does **not** have a reporting responsibility, you may contact the Office for Violence Prevention and Victim Assistance at (973)353-1918 or email run.vpva@rutgers.edu]

If students who have experienced a temporary condition or injury that is adversely affecting their ability to fully participate, you should submit a request via <https://temporaryconditions.rutgers.edu> .

If you are a military *veteran* or are on active military duty, you can obtain support through the Office of Veteran and Military Programs and Services. <http://veterans.rutgers.edu/>

If you are in need of *mental health* services, please use our readily available services.

[Rutgers University-Newark Counseling Center: <http://counseling.newark.rutgers.edu/>]

[Rutgers Counseling and Psychological Services–New Brunswick: <http://rhscaps.rutgers.edu/>]

If you are in need of *physical health* services, please use our readily available services.

[Rutgers Health Services – Newark: <http://health.newark.rutgers.edu/>]

[Rutgers Health Services – New Brunswick: <http://health.rutgers.edu/>]

If you are in need of *legal* services, please use our readily available services: <http://rusls.rutgers.edu/>

Students experiencing difficulty in courses due to *English as a second language (ESL)* should contact the Program in American Language Studies for supports.

[Rutgers–Newark: PALS@newark.rutgers.edu]

[Rutgers–New Brunswick: eslpals@english.rutgers.edu]

If you are in need of additional *academic assistance*, please use our readily available services.

[Rutgers University-Newark Learning Center: <http://www.ncas.rutgers.edu/rlc>

[Rutgers University-Newark Writing Center: <http://www.ncas.rutgers.edu/writingcenter>]

[Rutgers University-New Brunswick Learning Center: <https://rlc.rutgers.edu/>]

[Optional items that many faculty include:

- Students must sign, date, and return a statement declaring that they understand the RU Academic Integrity Policy.

- Students must sign, date, and return a statement declaring that they understand this syllabus.]