

Operation Research
COURSE NUMBER: 26:711:651
COURSE TITLE: Linear Programming

COURSE DESCRIPTION

Our main purpose is to cover a broad spectrum of topics in linear programming. We cover the basic material such as the simplex method and duality theory, and provide introduction to related techniques, such as dynamic programming and integer programming. A good part of the course will be devoted to the modeling power of linear programming and its applications particularly in business disciplines such as supply chains, finance, marketing etc. We will also introduce the modeling language AMPL for building LP models for simple applications, and use a related optimization package to solve small numerical examples.

COURSE MATERIALS

Recommended text books:

- *Linear Programming: Foundations and Extensions* by Robert J. Vanderbei (available for download: <http://www.princeton.edu/~rvdb/LPbook/index.html>)
- *Linear Programming* by Vašek Chvátal (W.H. Freeman and Company, 1980) ,
- *AMPL: A Modeling Language for Mathematical Programming* by Robert Fourer, David M. Gay, and Brian W. Kernighan (available for download: <http://ampl.com/resources/the-ampl-book/chapter-downloads/>)

Prerequisites:

Basic knowledge of linear algebra (solving systems of linear equations) and calculus.

No computer programming knowledge is needed.

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COURSE SCHEDULE

January 26, 2015: Introduction to linear programming, modeling languages, and tools. A simple example: Gepetto's wood carving shop. Geometric analysis. AMPL model of the problem. Basics of AMPL modeling. AMPL interfaces.

February 2, 2015: Review of linear systems of equations, matrix algebra, vector spaces, convexity, polytopes and polyhedra. Caratheodory's and Helly's theorems.1

February 9, 2015: The simplex algorithm and its interpretations; tableaus and dictionaries. Pitfalls and cures: feasibility, boundedness and finiteness.

February 16, 2015: The two phase revised simplex algorithm. Applications: blending, diet planning, scheduling.

February 23, 2015: The Fundamental Theorem of Linear Programming. Measuring efficiency; handling degeneracy; lexicographic rules; LU-factorization.

March 2, 2015: Weak and strong duality; complementary slackness. The Duality Theorem. Farkas' lemma and a proof of strong duality.

March 9, 2015: Sensitivity analysis and interpretation of duality in terms of input-output models. Regression theory: L_1 and L_∞ regression models; extracting trends, and modeling qualitative relations.

March 16, 2015: **Spring Break!**

March 23, 2015: Matrix games, Nash-equilibrium, connections to LP-duality, min-max theorems.

March 30, 2015: Assignment problems; primal-dual algorithms; the Hungarian method. Transportation problems and the primal-dual method.

April 6, 2015: Transshipment problems. Basics of networks and graphs; paths, cycles and spanning trees. The network simplex algorithm.

April 13, 2015: Network flows, circulations; the law of conservation. Graphical min-max relations. Integrality property; the max-flow-min-cut theorem.

April 20, 2015: Dynamic programming; Dijkstra's shortest path algorithm; Ford-Fulkerson algorithm for max-flows.

April 27, 2015: Knapsack problems, and the role of integrality; multicommodity flows. The basics of branch-and-bound for integer programming.

May 4, 2015: Dynamic programming for knapsack problems; approximation algorithms.

May 12, 2015: **Final Exam!**